Statistical dependence between first and second-order PMD

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Abstract: We measure the first and the second-order PMD in high-PMD fiber, and determine correlation between the first and second-order PMD. Theory and measurements are in a good agreement.

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1. Introduction

One of the difficulties in compensating PMD is caused by its statistical nature. In order to compensate for PMD, it is essential to know the statistical characteristics of the different PMD parameters. Such consideration must include not only first-order, but also higher-orders of PMD. Much of this information is now available. Statistics characterization, including probability density functions (PDF) of the second-order PMD (SOPMD) is given in [1,2]. However, one of the most important statistical characteristics, namely the statistical dependence between first and second order PMD has not yet been fully investigated.

As shown in [3] an angular speed of the rotation of principal states decreases with the value of differential group delay (DGD). The assumption was made that high values of the product of DGD on the angular speed (i.e. the perpendicular component of SOPMD) seldom occur when DGD is large and SOPMD is not important for large DGD. Our results show, that although the angular speed of the PSP rotation decreases with DGD, the product DGD*angular speed, is indeed, linearly increasing with DGD. Therefore, the penalty due to the uncompensated high-order PMD is larger for big DGD.

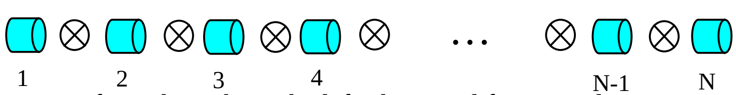
In this paper we follow a definition of two second-order components of SOPMD [4]: component parallel to PMD vector, called polarization dependent chromatic dispersion (PCD), and perpendicular to the PMD vector component, causing depolarization. We investigate the statistics of both components of SOPMD vector as functions of DGD. We demonstrate theoretically and experimentally, that the root mean square (rms) of the perpendicular component of SOPMD vector increases close to linear with DGD, while the rms of PCD does not depend on DGD.

2. Analytical approach

Our theoretical investigation of the PMD statistics we base on the expression for SOPMD vector of two concatenated fibers (see for example [4]):

$$\vec{\tau}_{\omega} = \vec{\tau}_{2\omega} + R_2\vec{\tau}_{1\omega} + \vec{\tau}_2 \times R_2\vec{\tau}_1 \qquad (1)$$

Here $\vec{\tau}_{\omega}$ is SOPMD vector of two concatenated pieces of fiber, R_2 is the transmission matrix of the second fiber, $\vec{\tau}_1$, $\vec{\tau}_2$ $\vec{\tau}_{2\omega}$, and $\vec{\tau}_{2\omega}$ are the first and second order PMD vectors of the first and the second sections, respectively. We represent a PMD fiber as a combination of N sections of polarization-maintaining fiber with random polarization scattering between the sections.



Light in this picture goes from the right to the left: this simplifies consideration. The cylinders represent pieces of birefringent fiber and the crossed circles represent random polarization scattering between the sections described by the random matrix *R*. Here we assume that *R* provides uniform scattering over the Poincaré sphere. Equation (1) used N times gives the following value for SOPMD vector:

$$\vec{\tau}_{\omega} = \sum_{n=1}^{N} \vec{\tau}_n \times R\Delta\vec{\tau} \qquad (2)$$

where $\Delta\vec{\tau}$ is the PMD vector of a single section and $\vec{\tau}_n$ is the PMD vector of n concatenated sections.

Equation (2) is a sum of cross products of randomly oriented vector $R\Delta\vec{\tau}$ with the PMD vector of *n* sections $\vec{\tau}_n$. The last terms in the sum (2) are highly correlated with the vector $\vec{\tau}$ of the entire link and, therefore, the cross products $\vec{\tau}_n \times R\Delta\vec{\tau}$ will be nearly perpendicular to the vector $\vec{\tau}$ for large *n*. The smaller becomes n the less correlation remains between $\vec{\tau}_n$ and $\vec{\tau}$. To within a good approximation we can assume that all the vectors $\vec{\tau}_n$ with the numbers n larger than a certain number K have the same magnitude and direction as $\vec{\tau}$ and all the vectors with smaller numbers are completely uncorrelated with $\vec{\tau}$. In this case the sum (2) will be divided into two parts one proportional to $\vec{\tau}$ and another with the direction random with respect to $\vec{\tau}_N$:

$$\vec{\tau}_\omega = (N-K)\vec{\tau}\Delta\tau + \sum_{n=1}^{K} \vec{\tau}_n \times R\Delta\vec{\tau} \qquad (3)$$

Squaring (3) and averaging we obtain the following expressions for the root mean squares of the parallel $\vec{\tau}_{\omega\parallel}$ and perpendicular $\vec{\tau}_{\omega\perp}$ components of $\vec{\tau}_\omega$:

$$\sqrt{<\vec{\tau}^2_{\omega\perp} \mid \tau>} = \frac{\sqrt{2}}{3}\sqrt{<\vec{\tau}^2>\tau^2 + \frac{1}{3}<\vec{\tau}^2>^2} \quad \text{and} \quad \sqrt{<\vec{\tau}^2_{\omega\parallel} \mid \tau>} = \frac{1}{\sqrt{27}}<\vec{\tau}^2> \qquad (4)$$

Here the sign $<\ |\tau>$ denotes averaging over all states with fixed τ, while <> denotes averaging over all possible states with all possible τ.

Because $<\vec{\tau}^2_{\omega\parallel}>$ does not depend on the value τ, the average PCD with fixed τ is the same with the average PCD including all possible τ : $<\vec{\tau}^2_{\omega\parallel} \mid \tau> = <\vec{\tau}^2_{\omega\parallel}>$. Therefore, in deriving (3) we were able to use a known relationship for the PCD [2]. Numerical calculations based on 10^6 samples are in excellent agreement with (4).

3. Experimental Results

The test fiber consisted of 12km of high PMD Dispersion Compensating Fiber (DCF) and 50km standard Corning SMF-28 fiber. To reduce the negative Chromatic Dispersion (CD) of the DCF 50km of SMF is used. The total CD of the PMDE was measured to be –246ps/nm.

We measured DGD and PSP as functions of wavelength using both MMM [5] and JME [6] methods. We found out that in the presence of PDL three-launch JME method gives more accurate results than two-launch MMM method. Using the measured values of DGD and PSP, we calculated both PCD and $\vec{\tau}_{\omega\perp}$.

Shown in Fig.1a is a histogram of the DGD. The histograms of PCD component, $\tau_{\omega\parallel}$ and absolute value of perpendicular component $\vec{\tau}_{\omega\perp}$ are presented in Fig.1b,c. The analytical curves were calculated directly from theoretical model [2] using the value of $\langle\tau\rangle = 41\text{ps}$ obtained from DGD measurements.

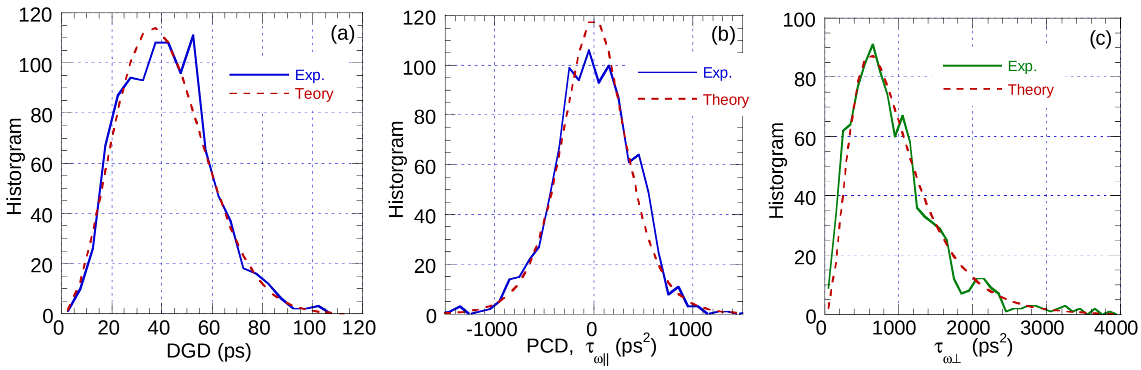


Fig. 1. Histograms of the DGD, PCD, and $|\vec{\tau}_{\omega\perp}|$.

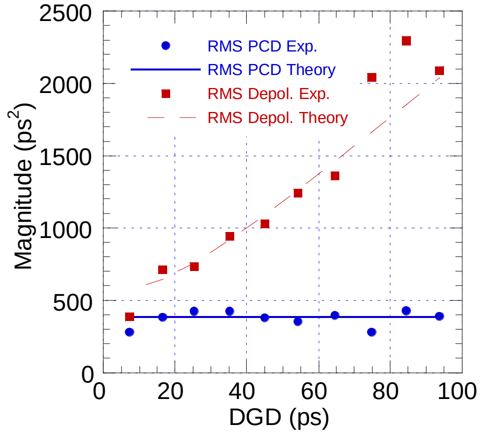


Fig.2. Dependence of rms of PCD and $\vec{\tau}_{\omega\perp}$ on DGD.

The main result of our investigation is shown in Fig 2. It represents the statistical dependence of the parallel (PCD) and perpendicular $\vec{\tau}_{\omega\perp}$ components of the second-order PMD vector on DGD. We averaged the DGD values within 10 ps size bins, and calculated corresponding rms for PCD and $\vec{\tau}_{\omega\perp}$ in the same bins. Overall we used 1000 experimental points. Surprisingly enough, the two components of the second-order vector have very different behavior with respect to the DGD in the line. The rms of the PCD does not depend on DGD, while $\vec{\tau}_{\omega\perp}$ for big DGD increases linearly. The deviation of the experiment from the theory for the large and small DGD values, we attribute to insufficient statistics in the tails of Maxwellian distribution.

4. Conclusions

We measured the first and the second-order PMD in a real fiber. Experimental and the theoretical data shows that the root mean square value of PCD does not depend on DGD. At the same time the root mean square of the perpendicular component $\vec{\tau}_{\omega\perp}$ increases linearly with DGD. Therefore the importance of the second and higher-order effects increases with the first-order DGD. There is an excellent agreement between the theory and experiment.

References

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